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MODELING OF GAS CENTRIFUGAL SUSPENSIONS SEPARATION PROCESSES

We consider the phase separation in the suspension krupnodispersnoy tsenrobezhnyh strength of the vortex chamber. The mathematical model and studied the hydrodynamic characteristics of gas flow, a film of viscous liquid on a cylindrical permeable surface of the swirl chamber and the motion of solid particles. In a cylindrical coordinate system to obtain an exact self-similar solution for the radial and tangential velocity components of the liquid phase because of its tangential stresses for a given friction at the interface and the pressure of the gas stream. The analysis of the hydrodynamic characteristics of the film movement in the permeable surface area calculation and filtering performance of the solid phase for the experimental setup.

Introduction. Separation of multiphase systems is a component of many industrial processes in various industries. Different vortex devices are used to intensify separation processes. These devices have simple structure, low steel intensity and hydraulic resistance. At the same time vortex devices work in a wide range of loading variations for liquid and gas [1]. Engineering and implementation of devices require mathematical modeling of the separation processes.

The process of separating coarse suspensions on cylindrical surface by swirling gas stream formed by a rotating device (rotor of a fan), was studied on the created installation (Fig. 1). A cylindrical perforated element *2* with larger diameter located in a body *3* with an inlet branch for suspension *4* and an exit branch for solid phase *5* is installled in alignment of rotor *1*.

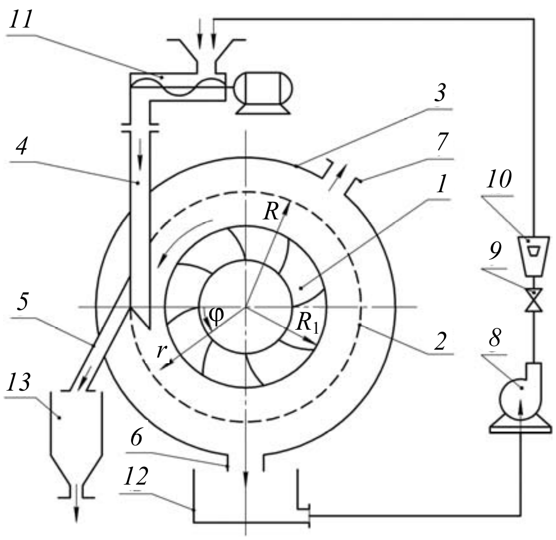

Fig. 1. Experimental device:
1 – rotor; *2* – perforated element;
3 – body; *4* – inlet branch for the suspension;
5 – exit branch for the solid phase;
6 – exit branch for the liquid phase;
7 – exit branch for the gas; *8* – pump;
9 – valve; *10* – rotameter; *11* – screw feeder;
12 – vessel for the liquid;
13 – vessel for the solid phase

The input of a suspension and the output of the solid phase are in the same place to use the whole surface of the perforated element. Suspension filtration occurs on a certain segment of perforated surface, and the boundary-layer separation of liquid film at the rest of the surface.

Liquid dropped out of the body through the exit branch *6*. A device to control discharged gas was located in the exit branch *7*. The liquid was pumped *8* through a controlled valve *9* and rotameter *10* to measure its consumption. Screw feeder inputs the suspension. There are vessels *12*, *13* to collect separated liquid and solid phases.

Mathematical model. The hydrodynamics of many vortical chambers can be modelled as flat movement of streams between rotating coaxial nontight cylinders of infinite length [2]. The fan is study as internal.

The nontight cylinder in radius R_1, and the external nontight cylinder in radius R is motionless. On a site of film movement the external cylinder is considered by the impenetrable.

Let's choose cylindrical system of coordinates r, φ, z with an axis z on an axis of cylinders. We will designate through $\tilde{r} = r / R$ – relative radius, and $r_0 = R_1 / R$.

At linear speed of internal cylinder $W_1 = \pi n / 30$ and in the absence of outflow the tangential component of a gas stream is described by dependence [2]:

$$W_\varphi = \frac{W_1 r_0}{1 - r_0^2}\left(\frac{1}{\tilde{r}} - \tilde{r}\right). \tag{1}$$

Let's find tangents of voltage of forces of a friction between cylindrical layers of individual length:

$$\tau_{r\varphi} = \frac{\mu_G}{R}\left[\tilde{r}\frac{\partial}{\partial \tilde{r}}\left(\frac{W\varphi}{\tilde{r}}\right)\right] = -\frac{2\mu_G W_1 r_0}{R(1 - r_0^2)}\frac{1}{\tilde{r}^2}.$$

On border of section of phases at a small thickness of a film, accepting $\tilde{r} = 1$, we will receive

$$\tau_{r\varphi} = -\frac{2\mu_G W_1 r_0}{R(1 - r_0^2)}. \tag{2}$$

At known pressure P_0 upon the internal cylinder (rotor), integrating the equation

$$\frac{dP}{dr} = \rho_G \frac{W_\varphi^2}{r}. \tag{3}$$

Let's receive pressure of gas upon an external impenetrable cylindrical surface, or on a surface of a film of a liquid:

$$P_G = P_0 + \rho_G \int_{r_0}^{1} \frac{W_\varphi^2(\tilde{r})}{\tilde{r}} d\tilde{r} =$$

$$= P_0 + \rho_G \frac{W_1^2 r_0^2}{1 - r_0^2}\left(\frac{1}{2r_0^2} + \frac{2\ln r_0}{1 - r_0^2} + \frac{1}{2}\right). \tag{4}$$

The pressure difference created by the fan [3], is defined by dependence

$$P_0 = k\frac{\rho_G W_1^2}{2} \tag{5}$$

with factor $k = 0.9–1.1$.

Examine the filtering process in film movement of the liquid phase on a permeable surface in the vortex chamber.

Outflow speed determined by the properties of a continuous medium permeable surface and pressure drop on it. When turbulent motion of the medium through the holes in the wall on the basis of the Bernoulli equation are square law [4, 5]:

$$\Delta P = \zeta \frac{\rho U_0^2}{2}. \tag{6}$$

At the simplest part of the surface speed of the outflow U_0 be considered constant, and the movement of self, in which the velocity components depend only on the radius. In this case, the radial velocity in the film is $U_r = U_0 R / r$.

Outflow and curvilinear forms a permeable surface have a stabilizing effect on the laminar boundary layer, reduce the degree of turbulence, significantly increases the friction with the surface, increase the limit of stability of laminar motion [6]. The Navier – Stokes laminar self-similar motion of a viscous fluid in cylindrical coordinates [7] are converted to the form:

$$\frac{d^2U\varphi}{dr^2} - \frac{1}{r}(\alpha - 1)\frac{dU\varphi}{dr} - \frac{1}{r^2}(\alpha + 1)U_\varphi + \frac{g_\varphi}{\nu} = 0;$$

$$\frac{dP}{dr} = \rho_L\left(\frac{U_\varphi^2}{r} + \frac{U_0^2R^2}{r^3}\right) + \rho_L g_r. \tag{7}$$

Here $\alpha = U_0 R / \nu_L$ is the Reynolds number. For convenience we introduce the relative thickness of the film $\tilde{\delta} = \delta / R$ and get the general solution of equation (7):

$$U_\varphi(r) = \frac{c_1}{Rr} + \frac{c_2}{R} r^{\alpha+1} + \frac{g\cos\varphi\, R^2 r^2}{3(U_0 R - \nu_L)}. \tag{8}$$

For the boundary conditions take the adhesion condition on a permeable surface and equality of the tangential stress at the interface:

$$U_\varphi\big|_{r=R} = 0; \quad \frac{\mu_L}{R}\left[r\frac{\partial}{\partial \tilde{r}}\left(\frac{U_\varphi}{\tilde{r}}\right)\right] = -\tau_{r\varphi}. \tag{9}$$

If we determine arbidrary constants from boundary conditions we get distribution of velocity tangent in the film of the liquid:

$$U_\varphi(\tilde{r}) = \left[\frac{g\cos\varphi\, R^2\tilde{r}^2}{(\alpha+2)(U_0R - \nu_L)} + \frac{\tau_{r\varphi}R}{\mu_L(\alpha+2)}\right]\times$$

$$\times\left(\frac{1}{\tilde{r}} + \tilde{r}^{\alpha+1}\right) + \frac{g\cos\varphi\, R^2\tilde{r}^2}{3(U_0R - \nu_L)}\left(\tilde{r}^2 - \frac{1}{\tilde{r}}\right). \tag{10}$$

Integrating (8), we will discover pressure difference on a nontight surface and taking into account (6) we will receive the equation for definition of a velocity of outflow of a liquid phase:

$$P_G + \rho_L \int_{1-\tilde{\delta}}^{1}\left(\frac{U_\varphi(\tilde{r})}{\tilde{r}} + \frac{U_0^2}{\tilde{r}^3}\right)d\tilde{r} + \rho_L g \sin\varphi\, R\tilde{\delta} =$$

$$= \zeta\frac{\rho_L U_0^2}{2}. \tag{11}$$

The specific expense of a liquid q on recording cut is from expression

$$R\int_{1-\tilde{\delta}}^{1} U_\varphi(\tilde{r})\, d\tilde{r} = q. \tag{12}$$

The modification of the specific expense is described by the equation

$$\frac{dq}{d\varphi} = -RU_0. \tag{13}$$

In a cylindrical frame we will write the equations of movement of a particle in radius an and a mass m under the influence of a rotational stream [7]. The scheme of operating forces is shown on Fig. 2, and their calculation is fulfilled in work [8]:

$$\begin{cases} m\left(\dfrac{dV_r}{dt} - \dfrac{V_\varphi^2}{r}\right) = F_r - F_L^N - mg\cos\varphi + F_A\cos\varphi, \\ m\left(\dfrac{dV_\varphi}{dt} + 2\dfrac{V_\varphi V_r}{r}\right) = F_\varphi + F_L^\varphi - F_F - \\ -\ mg\sin\varphi + F_A\sin\varphi, \\ I\dfrac{\partial\omega}{\partial t} = M_F + M_\tau. \end{cases}$$

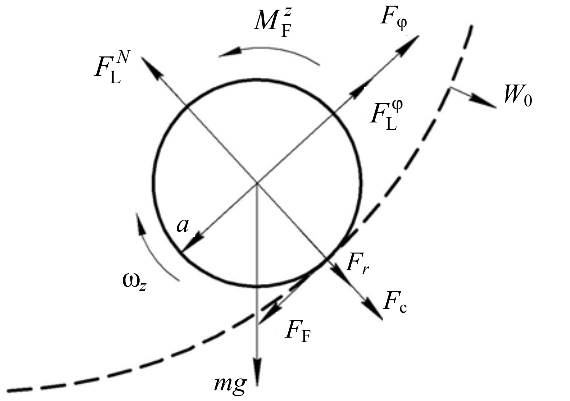

Fig. 2. The scheme of forces operating on a particle

The set of equations (11)–(13) is closed and defines hydrodynamic performances film of gas centrifugal filtering of a liquid. Its solution allows to calculate a step by step method optimum loading on a liquid phase and a site of filtering depending on geometrical and dynamic characteristics of a rotational means.

Results of calculations. For study of process of separation used suspensions of polymers: polyethylene – water and polystyrene – water. The polyethylene denseness made 930–970 kg/m^3, polystyrene – 1,000–1,050 kg/m^3.Volume concentration of a firm phase in suspension given on separation varied within 20–50%. Granules had the spherical form in diameter of 1–5 mm. Researches were spent on a cylindrical element in radius R = 0.22 m, length L = 0.01 m, with relative square of orifices f = 0.16, resistance ζ = 88.15. The suspension expense made Q = 0.1–1.0 m^3/h.

As the generator of the twirled gas stream the wheel in radius R_0 = 0.8 centrifugal ventilators served. Factor k = 0.9. Pressure and a stream velocity in the working channel governed by a modification of frequency of rotation of a wheel within 500–2,000 rev/min. The general expense of air thus made 35–100 m^3/h. Pressure and making velocities of a gas stream were measured by means of a three-channel probe and differential manometr. The amount of passable gas through a grid was measured by means of tubes of Pito [9] in branch pipes *5*, *6* and *7* and made 70% from its general expense (Fig. 1 see).

Productivity of a means is defined by loadings on a liquid phase and an outflow velocity through a nontight surface. The outflow velocity in turn depends on the pressure difference created by a gas stream and centrifugal forces of a moving recording.

At a solution of a set of equations (8)–(13) values have been received: c_1 = –0.61; c_2 = –0.32; U_0 = 0.17 km/s; δ = 0.013 m. The filtering site made no more than $^1/_4$ nontight elements.

The received mathematical model and the fulfilled calculations give the chance to analyse hydrodynamic performances of film movement on a nontight surface.

In Fig. 3 modifications of number of Reynolds and a thickness of a recording δ are reduced, and on Fig. 4 modifications of a velocity of filtering U_0 and mean value of a tangential making velocity of a recording of a liquid are shown at loading on a liquid phase 0.5 m^3/h and frequency of rotation of a curl of 1,500 rev/min. Calculation of a site of filtering of a liquid phase depending on frequency of rotation of a ventilator is presented in Fig. 5.

Performance measured under the condition that the particles move at one diameter apart. At this distance decay swirling flow, and therefore true formulas of motion [5].

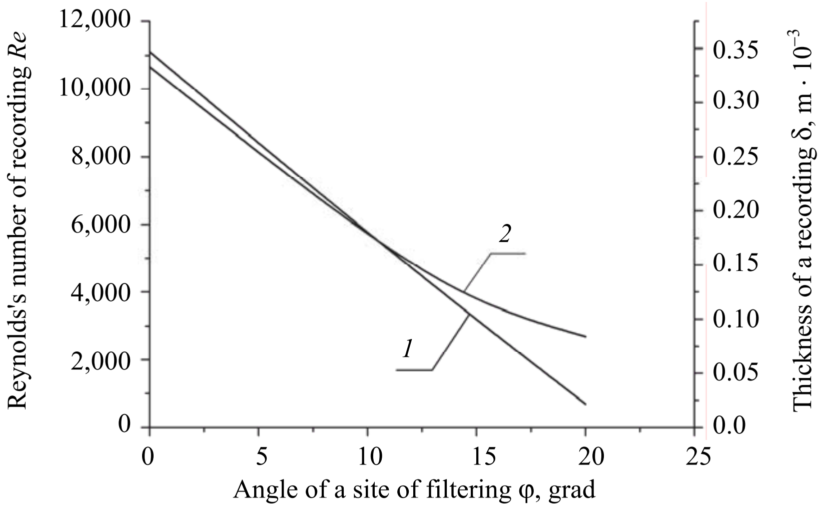

Fig. 3. Hydrodynamic performances of a recording of a liquid on a filtering site (n = 1,500 rev/min, Q = 0.5 m^3/h):
1 – liquid Reynolds's number of a recording;
2 – a thickness of a recording of a liquid $\delta \cdot 10^{-3}$, m

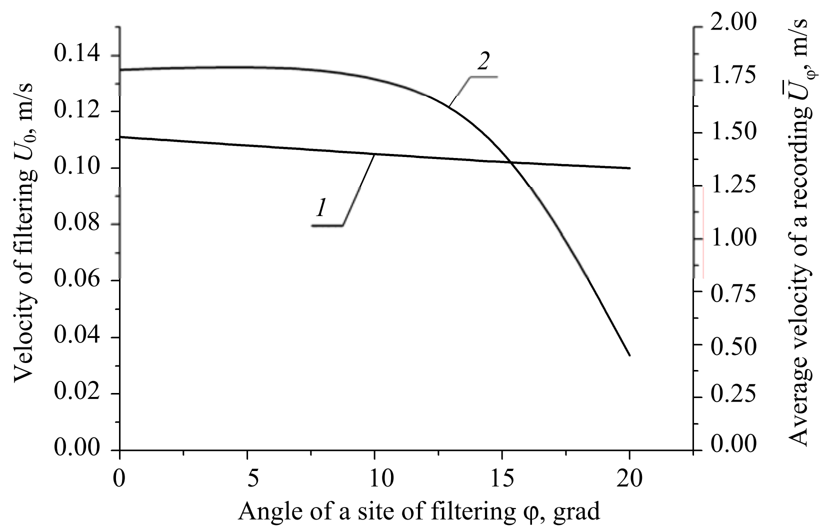

Fig. 4. Hydrodynamic performances of a recording of a liquid on a filtering site (n = 1,500 rev/min, Q = 0.5 m^3/h):
1 – a velocity of outflow of a liquid phase;
2 – an average velocity of a recording along a filtering surface

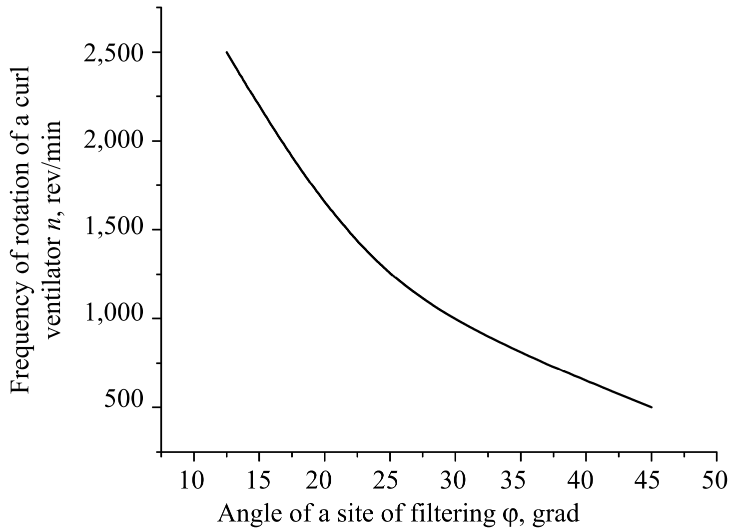

Fig. 5. Association of a site of filtering on frequency of rotation of a ventilator (Q = 0.5 m^3/h)

Speed of filtering of U_0 substantially is defined by pressure created by the twirled gas stream, and is almost constant. Its little change is characterized by the centrifugal force of a film of the liquid moving on a cylindrical surface.

As showed results of pilot studies, the liquid phase of the low-concentrated suspensions of polyethylene and polystyrene separated on the surface which isn't exceeding $^1/_5$ punched element, within change of specific loading of 6–100 $m^3/(m^2 \cdot h)$. On other surface there was a failure of a liquid film from a surface of particles. Thus the increase in speed of a gas stream only slightly influenced humidity of a firm phase. In studied operating modes of installation humidity of particles fluctuated within 0.7–1.1%.

Thus, the second task is ensuring steady transportation of particles on a surface of the punched element.

Calculation for the received models of a gas stream and movement of particles in the vortex camera, taking into account all operating forces, allows to define nature of movement of particles in the vortex camera.

Setting particle speed in a point of its initial situation, numerical methods counted a trajectory of movement and speed (Fig. 6).

Calculated data on the minimum gas velocity, providing particle removal from the device, confirmed by experimental studies. Sustainable transportation and high performance provided by a solid phase at a sufficiently low average velocity of the gas (see Fig. 5).

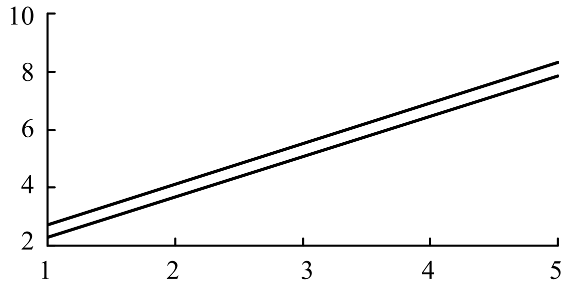

Fig. 6. Dependence of the minimum average tangent speed of gas $\bar{W}_\varphi$ (straight line *1*) and productivities on a firm phase *Q* (straight line *2*) from diameter of spherical particles

Plant capacity constraints limited specific load on the liquid phase up to 50 $m^3/(m^2 \cdot h)$. At higher flow observed accumulation of fluid in the bottom of the unit and a sharp increase in the moisture content of solid particles.

Conclusion. The mathematical models of gas flow in the vortex chamber, a film movement in the permeable surface under the influence of the gas flow and the movement of particles are elaborated. These models are take into account the interaction of the phases and the driving force of the process of filtering, the hydrodynamic characteristics are determined, load of the liquid and solid phases, depending on the geometrical parameters and operating modes of the device, are calculated.

The presented model can be used to calculate designs gas centrifugal apparatus for separating suspensions.

Notation. C_1, C_2 – the constant coefficients; f – relatively permeable surface area of the openings; g – acceleration of gravity, m/s^2; L – length of the permeable cylindrical element, m; n – frequency of rotation of the cylinder, rev/min; P, ΔP – pressure, pressure drop respectively, Pa; P_0 – pressure due to the rotating cylinder, Pa; k – factor; Q – volumetric flow rate of the liquid phase, m/h; q – specific flow rate of the liquid phase, $m^3/(m \cdot s)$; r – the distance in the radial direction in cylindrical coordinates, m; R – radius of the permeable cylindrical element, m; R_1 – radius of the rotating cylinder, m; $\tilde{r} = r / R$ – the dimensionless radial coordinate; $r_0 = R_1 / R$ – the ratio of the radiuses of the cylinders; Re – Reynolds number of the liquid film; U_φ, U_r – the tangential and radial velocity components of the liquid, respectively, m/s; W_1 – linear velocity of the rotating cylinder, m/s; W_φ, W_r – tangential and radial components of the gas flow rate, respectively, m/s; U_0 – the rate of outflow of fluid through the permeable surface, m/s; α – factor; z – axial coordinate of the cylindrical coordinate system, m; δ – thickness of the liquid film, m; $\delta = \delta / R$ – dimensionless thickness of the liquid film; $\pi = 3.14159...$; ζ – drag coefficient; μ – coefficient dynamic viscosity, N · s/m^2; ν – coefficient kinematic viscosity, m^2/s; ρ – density, kg/m^3; τ – shear stress, N/m^2; φ – angle in a cylindrical coordinate system.

Indexes. G – gas, L – liquid.

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